

Integration techniques - Parts and more!

Tabular method: Useful for multiple stages of integration by parts.

Ex.1 - Polynomials that disappear after differentiation

$$\int z \sin z \, dz =$$

signs	u and its derivatives	dv and its antiderivatives
+		
-		
+		

Ex.2 – Forms that reappear

$$\int e^{2x} \sin 3x \, dx =$$

signs	u and its derivatives	dv and its antiderivatives
+		
-		
+		

Calc 2

Try it yourself

a. $\int 2x^4 \cos x \, dx =$

signs	u and its derivatives	dv and its antiderivatives
+		
-		
+		
-		
+		
-		
+		

b. $\int (x^5 + 2x^3 - 8) e^{3x} \, dx =$

signs	u and its derivatives	dv and its antiderivatives
+		
-		
+		
-		
+		
-		
+		

Calc 2

c. $\int e^x \cos x \, dx =$

signs	u and its derivatives	dv and its antiderivatives
+		
-		
+		
-		
+		
-		
+		

Calc 2

Integration of trigonometric functions

What we already know...

$$\int \sin x \, dx =$$

$$\int \cos x \, dx =$$

$$\int \sec^2 x \, dx =$$

$$\int \csc^2 x \, dx =$$

$$\int \sec x \tan x \, dx =$$

$$\int \csc x \cot x \, dx =$$

What is missing?

Calc 2

1. One extra sine or cosine

Evaluate the following using u substitution techniques:

a. $\int \sin x \cos x \, dx =$

b. $\int \sin^2 x \cos x \, dx =$

c. $\int \sin^3 x \cos x \, dx =$

d. $\int \frac{\sin x}{\cos x} \, dx =$

e. $\int \frac{\sin x}{\cos^2 x} \, dx =$

Do you see a pattern?

Conclusions:

What if we switched sines and cosines?

Calc 2

Let's fill in some of the gaps now:

$$\int \tan x \, dx =$$

$$\int \cot x \, dx =$$

$$\int \sec x \, dx =$$

$$\int \csc x \, dx =$$

Calc 2

2. Only odd powers of sine (or cosine) – recall that $\sin^2 x + \cos^2 x = 1$.

$$\int \sin^3 x \, dx =$$

$$\int \cos^{2/3} x \sin^3 x \, dx =$$

3. Only even powers of cosine (or sine) – recall that $\cos^2 x = \frac{1+\cos 2x}{2}$ and $\sin^2 x = \frac{1-\cos 2x}{2}$

$$\int \cos^2 x \, dx =$$

$$\int \cos^4 x \, dx =$$

Combining things:

$$\int \sin^2 x \cos^2 x \, dx =$$

Calc 2

Practice:

Compute the following antiderivatives

1. $\int \sin^{4/5} x \cos x \, dx$

2. $\int \cos^3 x \sin x \, dx$

3. $\int \sin^5 x \, dx =$

4. $\int \cos^{1/5} x \sin^3 x \, dx$

5. $\int \cos^2 x \sin^4 x \, dx$