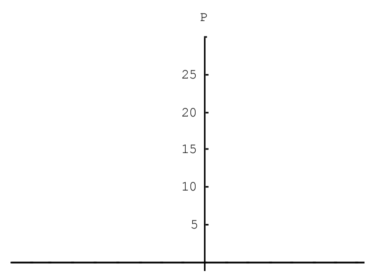
MT 3700 Differential Equations
Phase Lines & Classifying Equilibria

Name: ______________________________

1. Consider the differential equation $\frac{dP}{dt} = \left(1 - \frac{P}{20}\right)^3 \left(\frac{P}{5} - 1\right) P^7$.

   a. Draw the <u>phase line</u> for this differential equation.

   b. On the axes below, sketch the <u>particular solutions</u> to this differential equation satisfying the initial conditions: $P(0) = 5$, $P(0) = 10$, $P(0) = 20$, and $P(0) = 25$.



2. *Classifying Equilibrium Points*: There are four possibilities for equilibrium points, $y = y_e$ on a phase line:

(1)	(2)	(3)	(4)

- **Type (1):** This kind of equilibrium point is called a **sink**. Draw some sample solution curves above and below the sink, $y = y_e$ (assuming uniqueness). Notice that solutions with initial conditions close to a sink, $y = y_e$, tend *toward* $y = y_e$.

- **Type (2):** This kind of equilibrium point is called a **source**. Draw some sample solution curves above and below the source, $y = y_e$ (assuming uniqueness). Notice that solutions with initial conditions close to a source, $y = y_e$, tend *away from* $y = y_e$.

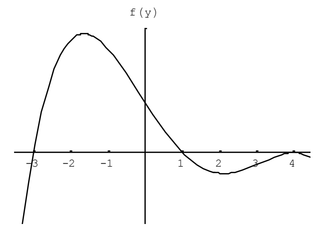
- **Types (3) and (4):** This kind of equilibrium point is called a **node**. Draw some sample solution curves above and below the node, $y = y_e$ (assuming uniqueness).

a. Consider the differential equation $\dfrac{dy}{dt} = y^2 - 2y - 3$.

i. Draw the phase line for this differential equation and classify the equilibrium points.

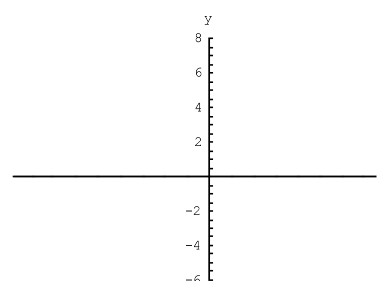
ii. What conclusions can you draw from the Uniqueness Theorem about a solution to this differential equation which satisfies the initial condition $y(0) = 1$? (Give bounds on this solution and long-term behavior.)

b. Drawing a Phase Line and Classifying Equilibrium Points from the graph of $f(y)$:
Consider the following graph of the function $f(y)$.



i. Draw the phase line (and classify the equilibrium points) for the differential equation $\frac{dy}{dt} = f(y)$.

ii. On the axes below, sketch the graphs of the solutions to this differential equation satisfying the initial conditions: $y(0) = -5$, $y(0) = -1$, $y(0) = 2$, & $y(0) = 7$.



c. Use your knowledge about derivatives from Calculus to complete the following theorem that gives analytic criteria specifying the type of equilibrium point:

Linearization Theorem: Suppose y_e is an equilibrium point of the differential equation $dy/dt = f(y)$, where f is a continuously differentiable function (i.e., f and f' are both continuous). Then,

- If $f'(y_e) < 0$, then y_e is a ________ (sink, source, or node).
- If $f'(y_e) > 0$, then y_e is a ________ (sink, source, or node).
- If $f'(y_e) = 0$, is y_e necessarily a node? Explain.